



GARISSA UNIVERSITY

**UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR TWO
SECOND SEMESTER EXAMINATION**

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCE

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

COURSE CODE: ACS 201

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS 1

EXAMINATION DURATION: 3 HOURS

DATE: 19/04/18

TIME: 09.00-12.00 PM

INSTRUCTION TO CANDIDATES

- The examination has SIX (6) questions
- Question ONE (1) is COMPULSORY
- Choose any other THREE (3) questions from the remaining FIVE (5) questions
- Use sketch diagrams to illustrate your answer whenever necessary
- Do not carry mobile phones or any other written materials in examination room
- Do not write on this paper

This paper consists of FOUR (4) printed pages

please turn over



QUESTION ONE (COMPULSORY)

- (a) Explain briefly the concept of selection in relation to mortality [2 marks]
- (b) A fund is earning 7% simple interest. Calculate the effective rate in the 8th year [3 marks]
- (c) A certain life table has a select period of 1 year. At each integer age X the select rate mortality is 50% of the ultimate rate. Calculate $e_{[60]}$ given that $e_{[60]} = 17.5424$ and $q_{60} = 0.0142$ [4 marks]
- (d) If a fund accumulates at force of interest $\delta(t) = 0.02t$. Find the annual effective rate of interest over 2 years and 5 years [4 marks]
- (e) On the basis of ELT No. 12 – Males find the probability that a life aged 30 will
- Survive to age 40 [1 mark]
 - Die before reaching age 50 [2 marks]
 - Die in his 50th year of age [2 marks]
 - Die between his 40th birthday and his 50th birthday [3 marks]
- (f) You are given that the nominal rate of discount per annum convertible every 3 months is 15%. Calculate the nominal rate of interest per annum convertible every 3 months. [4 marks]

QUESTION TWO

- (a) Construct an amortization schedule of a loan of 15,000 shillings to be paid over 6 years with a 6 payment annuity immediate at effective rate of interest of 6% per year [10 marks]
- (b) On April 1, company X purchased equipment for Kshs 100, 000. This is expected to have 5 useful life years. The salvage value is Kshs 14,000. Company X considers depreciation expense for the nearest whole month. Calculate the depreciation expense for 2012, 2013, and 2014 using declining balance method [5 marks]

QUESTION THREE

- (a) A company wants to provide a retirement plan for an employee who is aged 55 now. The plan will provide her with an annuity immediate of Shs 7,000 every year for 15 years upon her retirement at the age of 65. The company is funding this plan with an annuity due of 10 years. If the rate of interest is 5%. What is the amount of installment the company should pay [8 marks]
- (b) Payments are made to an account at continuous rate of $(8k + tk)$ where $0 \leq t \leq 10$. Interest is credited at a force of interest $\delta(t) = \frac{1}{8+t}$. After 10 years, the account is worth 21, 600. Calculate k . [7 marks]



QUESTION THREE

(a) A loan is to be repaid by an 8 payment annual annuity immediate of 200 shillings at an interest of 5% for both the loan charged and sinking fund credit.

- i. What is the interest component of the 4th payment in the amortization schedule [4 marks]
- ii. What is the interest credited to the sinking fund at the end of 5 year [4 marks]

(b) Brian can receive one of the following payment streams

- i. 100 at time 0, 200 at time n and 300 at time 2n
- ii. 600 at time 10

At an annual effective interest rate of i the present value of the two streams are equal. Given $V^n = 0.759412$, determine i [7 marks]

QUESTION FOUR

(a) For a certain population

$$l_x = \frac{l_o}{(1+x)^2}, \quad (x \geq 0)$$

Calculate

- i. The complete expectation of life at birth [2 marks]
 - ii. The force of mortality at age 1 [3 marks]
 - iii. The chance that a new born will die between ages 1 and 2 [3 marks]
- (b) Show that $tPx Ux + t$ is decreasing for $0 \leq t \leq 1$. Show that $qx < ux$ [3 marks]
- (c) Find the sum of the present value of two payment of Ksh 5000 to be paid at the end of 4 and 9
- i. If interest is compounded semiannually at the nominal rate of 8% per year [2 marks]
 - ii. The simple interest method of 8% [2 marks]

QUESTION FIVE

(a) Calculate $a_{\overline{n}|i}$ and $s_{\overline{n}|i}$ if the nominal rate of interest is 5% per annum assuming

- i. Compound interest [2 marks]
- ii. Simple interest [2 marks]

(b) Find the present value of an annuity immediate of 100 shillings for 4 years if interest is compounded semiannually at the nominal rate of 6% [5 marks]

(c) A Perpetuity costs 77 shillings and makes an annual payment at the end of the year. It pays 1 at the end of year 2, 2 at the end of year 3, ..., n at the end of year $n + 1$. After year $n + 1$, the payment remains constant at n. assuming an effective rate of 10.5% calculate n. [6 marks]



QUESTION SIX

- (a) Assuming an interest of 12% p.a convertible monthly,
- Calculate the combined present value of an immediate annuity payable monthly in arrears such that payments are Kshs 100, 000 p.a for the first 6 years and Ksh 40, 000 p.a for the next 4 years together **[3 marks]**
 - Calculate the amount of the level of annuity payable continuously for 10 years having the same present value of the annuity in (i) above **[3 marks]**
 - Calculate the accumulated value of the first 7 years of payment at the end of the 7th payment in (i) and (ii) **[4 marks]**
- (b) Show that, if Makeham's law holds for all ages greater than or equal to α , there are positive constants s and g such that

$${}_tP_x = S^t g^{x(c^t-1)} \text{ for } x \geq \alpha, t \geq 0 \quad \textbf{[5 marks]}$$

