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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2020/2021 ACADEMIC YEAR ONE**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**FOR THE DEGREE OF BACHELOR OF INFORMATION SCIENCE**

**COURSE CODE: STA 121**

**COURSE TITLE: PROBABILITY AND STATISTICS II**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 19/08/2021 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

**+**ete random variable, $X,$is given by $P\left(X=x\right)=kx^{2}$ for $x=0,1,2,3,4.$ Given that $k$ is a constant, find the value of $k$ and hence find the pdf of the random variable (5 marks)

1. The discrete random variable $T$, has the probability distribution as shown below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$T$$ | -3 | -2 | -1 | 0 | 1 |
| $$P(T=t)$$ | 0.1 | 0.25 | 0.3 | 0.15 | $$d$$ |

1. Find the value of $d$ (2 marks)
2. Find $ P(-3<T\leq -1)$ (1 mark)
3. Find $P(T\geq -1)$ (1 mark)
4. A discrete random variable $X,$ has the following probability distribution function

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | -1 | 0 | 1 |
| $$f(x)$$ | $$a$$ | $$b$$ | $$c$$ |

If $b=\frac{1}{2},$ determine $E(X^{2})$ (3 marks)

1. Let $X$ be a continuous random variable with pdf

 $f\left(x\right)=\left\{\begin{array}{c}\frac{x}{5}+k, 0\leq x\leq 3\\0 elsewhere\end{array}\right.$ Find the value of $k$ and hence compute $P(1\leq x\leq 2)$ (5 marks)

1. A random variable $X$ can assume only odd integer values between 0 and 12. It is distributed in such a way that

 $f\left(x\right)=P\left(X=x\right)=\frac{x}{Σx}$

1. Find the probability distribution of $X$ (2 marks)
2. Find the $E(X)$ and the $Var(X)$ (5 marks)
3. The number of calls per 10 minutes received at safaricom switchboard follows a Poisson distribution with mean 0.6. Find the probability that
4. No calls will be received in the first 10 minutes (2 marks)
5. More than 2 calls will be received in a period of 40 minutes (4 marks)

**QUESTION TWO (20 MARKS)**

1. A random variable $X$ has the following probability distribution function

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $$f(x)$$ | 0 | $$k$$ | $$2k$$ | $$2k$$ | $$3k$$ | $$k^{2}$$ | $$2k^{2}$$ | $$7k^{2}+k$$ |

1. Find $k$ and complete the table (6 marks)
2. Determine the cumulative distribution function of $X$ (2 marks)
3. A continuous random variable $X$ has probability density function given by $f\left(x\right)=\left\{\begin{array}{c}kx^{2}, 1<x<3\\0, otherwise\end{array}\right.$ . Determine
4. The value of $k$ (2 marks)
5. $P(X<2)$ (4 marks)
6. The assembly time for a racing toy car manufactured by CMC follows a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5pm daily. If one worker starts assembling a racing car at 4 pm, what is the probability that he will finish the job before the company closes for the day? (6 marks)

**QUESTION THREE (20 MARKS)**

1. A random variable $X $ takes the values 1,2,3,4,5,6,7 which are mutually exclusive and mutually likely. Obtain the upper bound of

$$P\{\left|X-4\right|\geq 3\}$$

What is the exact probability? (11 marks)

1. If $X$ is the number scored in a throw of a fair die, show that the Chebychev’s inequality gives

$P\left[\left|X-E\left(X\right)\right|>2.5\right]<0.47$ (9 marks)

**QUESTION FOUR (20 MARKS)**

1. A continuous random variable $X$ has the probability distribution function given by $f\left(x\right)=\left\{\begin{array}{c}2x, 0\leq x\leq 1\\0, otherwise\end{array}\right.$ . Find the moment generating function of $X$ and use it to find $E(X)$ and $Var(X)$ (8 marks)
2. An airline knows that overall, 15% of the passengers do not turn up for flight. The airline decides to adopt a policy of selling more tickets than there are seats on a flight. For an aircraft with 11

seats, the airline sold 15 tickets for a particular flight. Obtain a suitable model for the for the number of passengers who do not turn up for the flight after buying a ticket

Find the probability that

1. More than 12 passengers turn up for this flight
2. Exactly 2 passengers miss seats on this flight
3. There are two empty seats on this flight. (12 marks)

**QUESTION FIVE 20**

1. A random variable, $X,$ has *pdf,* given as $f\left(x\right)=\left\{\begin{array}{c}6x\left(1-x\right), 0<x<1\\0, elsewhere\end{array}\right.$ find the *pdf* of $Y=X^{3}$ (5 marks)
2. Suppose that $X$ has a distribution given by

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$X$$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| $$f(x)$$ | $$\frac{4}{21}$$ | $$\frac{1}{6}$$ | $$\frac{1}{14}$$ | $$\frac{1}{7}$$ | $$\frac{1}{14}$$ | $$\frac{1}{6}$$ | $$\frac{4}{21}$$ |

Find the distribution of a random variable $u=3x^{2}+1$ (5 marks)

1. Let $X and Y $ be independent random variables each having probability function $f\left(x\right)=\left\{\begin{array}{c}e^{-x}, x>0\\0, elsewhere\end{array}\right.$ . Define $U=X+Y $ and $V=\frac{X}{X+Y}$

Find the joint *pdf* of $U and V $using the transformation technique (10 marks)