Ser. No. EDU 065/17

GARISSA UNIVERSITY

UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR TWO FIRST SEMESTER EXAMINATION

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA I

EXAMINATION DURATION: 3 HOURS

DATE: 08/12/17 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of TWO (2) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Define the following terms/phrases as used in the Linear Algebra:

i. A reduced echelon form matrix **[1 mark]**

ii. An elementary matrix **[1 mark]**

iii. The WROSKIAN of functions f,g,h.....at a point x_0 **[1 mark]**

iv. A basis S of a vector space V **[1 mark]**

v. A linear transformation T. **[1 mark]**

vi. A symmetric matrix. Give an example **[1 mark]**

(b) Given the matrix $A = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix}$

Where a, b, c and d are non-zero constraints,

i. Compute AA^t and A^tA. Hence determine A^{-1}, the inverse of A.

ii. Compute $\det A \times \det A^t$ **[7 marks]**

(c) Let $W = \{[\,4,5,6], [7,8,8]\}$ be a subset of $\mathbb{R}^3$

i. Show that W is linearly independent

ii. Describe the span of W

iii. Show that the span (W) is a subspace of $\mathbb{R}^3$ **[6 marks]**

(d) Row reduce to echelon form the augmented matrix

$$\left.\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{matrix}\right| \begin{matrix} a \\ b \\ c \end{matrix}$$

and obtain rank of A and inverse of A where $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{pmatrix}$ **[6 marks]**

QUESTION TWO

(a) (i) Let V be a vector space over a field $\mathbb{R}$. When are the vectors $v_{1,}v_{2,} \dots v_m, \in, V$ said to be linearly dependent (or independent) over $\mathbb{R}$? **[2 marks]**

(ii) Prove that the non-zero vectors $v_{1,}v_{2,} \dots v_m$ are linearly dependent if and only if one of them, say $v_{1,}$ is a linear combination of the preceding vectors:

$v_i = a_1 v_1 + \dots a_{i-1} v_{i-1}$ **[6 marks]**

(b) Let V be the vector space of a polynomials of degree ≤ 3over$\mathbb{R}$. Determine whether $u, v, w, \in V$ are independent or dependent where,

$u = t^3 + 4t^2 - 2t + 3, v = t^3 + 6t^2 - t + 4, w = 3t^3 + 8t^2 - 8t + 7$ **[7 marks]**

QUESTION THREE

(a) Suppose $U = \{sin\ x, sin\ 2x, sin\ 3x\}$ is the set of real valued functions of x.

i. Show that $W(\pi/4) = 1; W(x)$ is the Wroskianof$U$. Hence or otherwise, prove that U is linearly independent. **[5 marks]**

ii. If U, V are linearly independent vectors in a vector space y, then prove that the set $\{U + V, 2U - 3V\}$is linearly independent. **[4 marks]**

(b) A linear transformation T is defined on the vector space of$\mathbb{R}^2$ by $T[x, y] = [\ 2x, -y]$.

i. give a rule for the inverse transformation T^{-1} **[3 marks]**

ii. Obtain $T^{-1}[\ 2b, -c]$ **[1 marks]**

iii. Show that T^{-1} is linear **[2 marks]**

QUESTION FOUR

(a) Prove that for any vectors$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbb{R}^n$ and any scalar $\alpha \in \mathbb{R}^n$

$(\boldsymbol{x} + \boldsymbol{y})\boldsymbol{z} = \boldsymbol{xz} + \boldsymbol{yz}$ **[3 marks]**

(b) Let $V = \mathbb{R}^3$ .Show that W is a subspace V where $W = \{(a, b, c): a + b + c = 0\}$ that is W consists of those vectors each with the property that the sum of its components is zero.

[4 marks]

(c) Determine the three square elementary matrices corresponding to the operations $R_1 \leftrightarrow R_2$,$R_3 \leftrightarrow -7R_3$ and $R_2 \leftrightarrow -3R_1 + R_2$ **[3 marks]**

(d) Solve the system of equations: $2x_1 + x_2 - 2x_3 = 10$

$$3x_1 + 2x_2 + 2x_3 = 1$$

$$5x_1 + 4x_2 + 3x_3 = 4$$

by reducing the corresponding to an echelon form **[5 marks]**

QUESTION FIVE

(a) (i) Define the rank of a matrix **[2 marks]**

(ii) Using (i) above determine the rank of:

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$ **[3 marks]**

(b) Use triangular decomposition method to solve the system:

$$x_1 + 2x_2 - x_3 + 3x_4 = 10$$

$$2x_1 - x_2 + 3x_3 + 2x_4 = 23$$

$$3x_1 + 3x_2 + x_3 + x_4 = 5$$

$$4x_1 + 5x_2 - 2x_3 + 2x_4 = -2$$ **[10 marks]**

QUESTION SIX

Consider the transformation defined from $P_3(x)$into$P_3(x)$, the set of polynomials in degree 3 by:

$T(P_3(x)) = (x-1)P_3'(x) + P_3(x)$

i. Show that T is linear **[2 marks]**

ii. Obtain M_T, matrix of T **[3 marks]**

iii. Show that T is one – to – one and onto **[3 marks]**

iv. Obtain the kernel of T **[1 mark]**

v. Obtain the range of T **[3 marks]**

vi. Verify that Dim (Ker T) + Dim (range T) = Dim ($P_3(x)$). **[2 marks]**

vii. Obtain T^{-1} the inverse of T **[1 mark]**