Ser. No. EDU 069/17

GARISSA UNIVERSITY

UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR ONE FIRST SEMESTER EXAMINATION

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: MAT 104

COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

EXAMINATION DURATION: 3 HOURS

DATE: 11/12/17 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of THREE (3) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) State whether we can find a circle that passes through the points $A(1,2), B(2,4)$ and $C(5,6)$. **[5 Marks]**

(b) Solve for x between 0^0 and 360^0 in the equation $2sinx = cos(x+60°)$ **[5 Marks]**

(c) Find the roots of the equation $4x^4 - 19x^3 + 24x^2 + x - 10 = 0$. **[5 Marks]**

(d) (i) Find the eccentricity of the hyperbola $12x^2 - 27y^2 = 108$. **[2 Marks]**

(ii) Replace the following polar equation by its equivalent Cartesian equation and identify its graph: $r^2 = 4rcos\theta$. **[3 Marks]**

(e) (i) In how many ways can the letters of the word "ASSASSINATION" be arranged **[2 Marks]**

(ii) If $C(n,x) = 56$ and $P(n,x) = 336$ find n and x. **[3 Marks]**

QUESTION TWO

(a) Analyze the graph of the equation $9x^2 - 16y^2 - 144 = 0$. **[5 Marks]**

(b) Prove that the standard form of an equation of an ellipse, with centre (h,k) and major and minor axes of lengths $2a$ and $2b$ respectively, where $a > b$ is given by $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. **[10 marks]**

QUESTION THREE

(a) (i) State without proof, the remainder theorem **[1 Mark]**

(ii) Show that $\frac{3}{2}$ is a zero of $f(x) = 2x^3 - 5x^2 + x + 3$ and write $2x^3 - 5x^2 + x + 3$ in factored form. **[3 Marks]**

(b) (i) 4 men and 3 women are to be seated for a dinner such that no 2 women sit together and no 2 men sit together. Find the number of ways in which this can be arranged **[3 Marks]**

(ii) Verify that $\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 - 1$. **[3 Marks]**

(c) Show that the distance of a point $P(x_1, y_1)$ to a line $ax + by + c = 0$ in a Cartesian plane is given by:

$$r = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|.$$ **[5 Marks]**

QUESTION FOUR

(a) Prove the Binomial Theorem$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r} a^{n-r}b^r$. **[6 Marks]**

(b) Show that $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$. **[2 Marks]**

(c) Find a complete graph of $r = \frac{6}{4-3cos\theta}$ .Specify a directrix and a range for θ that produces a complete graph. Find the standard form for the equation of the conic. **[7 Marks]**

QUESTION FIVE

(a) Solve the following equations using the method indicated in brackets:
   (i) $Cos6\theta + Cos4\theta + Cos2\theta = 0for0° \leq \theta \leq 360°$ [Factor Formula]. **[4 Marks]**
   (ii) $4Cos\theta - 6Sin\theta = 5$ for$0° \leq \theta \leq 360°$
   [Rewrite in the form $Cos(\theta + \alpha) = C$]. **[4 Marks]**

(b) Prove that $\frac{tanx+secx}{secx\left(1+\frac{tanx}{secx}\right)} = 1$ by first rewriting each of the term in form of$sinx$, $cosx$or both. **[3 Marks]**

(c) Verify that the point (3, 2) lies on the circle $x^2 + y^2 - 8x + 2y + 7 = 0$ and find the equation of the tangent at this point. **[4 Marks]**

QUESTION SIX

(a) Prove that $Cosh\theta Cosh\phi - Sinh\theta Sinh\phi = Cosh(\theta - \phi)$. **[3 Marks]**

(b) If $5e^x - 2e^{-x} \equiv ASinhx + BCoshx$ find the values of A and B . **[4 Marks]**

(c) Solve the equation$3Coshx + 2Sinhx = 14.31$correct to 4d.p. **[4 Marks]**

(d) Obtain the first four terms of the expansion of $(1-16x)^{1/4}$.Substitute $x = \frac{1}{10000}$ and use the first two terms to find$\sqrt[4]{39}$. How many significant figures is your answer accurate? **[4 Marks]**