GARISSA UNIVERSITY

UNIVERSITY EXAMINATION **2017/2018** ACADEMIC YEAR **TWO SECOND** SEMESTER EXAMINATION

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: STA 211

COURSE TITLE: PROBABILITY AND STATISTICS II

EXAMINATION DURATION: 3 HOURS

DATE: 18/04/18 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of FOUR (4) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) A continuous random variable, X, has probability density function, $f(x)$, given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 10 \\ 0 & otherwise \end{cases}$$. Find

i. The value of k **[2 marks]**

ii. $P(X > 6)$ **[2 marks]**

iii. Calculate the exact value of $E(X)$ and $Var(X)$ **[2 marks]**

(b) A bag contains a large number of ksh 5 coins and ksh 10 coins in the ratio 1:3.

i. Find the mean, μ, and the variance, σ^2, of this population of coins **[3 marks]**

ii. Three coins are picked at random. Determine the probability distribution of the variable Y where Y is "the number of sh 10 coins chosen". **[3 marks]**

(c) The random variable Khas a binomial distribution with parameters $n = 25$ and $p = 0.27$. Find $P(K \leq 1)$ **[2 marks]**

(d) Find the moment generating function (MGF) of a random variable, X,which has a probability distribution given by $f(x) = \begin{cases} \frac{1}{8}\binom{3}{x} & for\ x = 0,1,2\ and\ 3 \\ 0 & otherwise \end{cases}$ and use the MGF obtained to determine the $Var(X)$ **[4 marks]**

(e) The growth of a sunflower plant is found to be normally distributed with a mean of 10 and a variance of 7.5. Find the probability that a sunflower picked at random will have a height between $8m$ and $13m$ (inclusive). **[4 marks]**

(f) Let x and y be jointly distributed random variables with joint probability density function $f(x) = \begin{cases} x + y & 0 < x\ 1, 0 < y < 1 \\ 0 & otherwise \end{cases}$ Determine whether x and y are independent

[3 marks]

QUESTION TWO

a) X is a random variable with the probability distribution, $f(x)$, given by $f(x) = \begin{cases} \frac{x+1}{20} & for\ x = 1,2,3,4,5 \\ 0 & otherwise \end{cases}$ Find the

i. Distribution function of X **[2 marks]**

ii. $P(2 \leq X < 4)$ **[1 mark]**

b) Two discs are drawn at random, without replacement, from a box containing 3 red discs and 4 white discs. If X is the random variable "$number\ of\ red\ discs\ drawn$", find

i. The expected number of red discs **[2 marks]**

ii. The standard deviation of X. **[2 marks]**

c) A continuous random variable, X, has the probability density function $f(x)$ given by $f(x) = \begin{cases} kx & for\ \ 0 \leq x < 2 \\ \frac{1}{2}kx(4 - x) & for\ \ 2 \leq x \leq 4 \\ 0 & otherwise \end{cases}$ where k is a constant. Find the

i. Value of k **[2 marks]**

ii. $E(X)$ and $Var(X)$ **[6 marks]**

QUESTION THREE

(a) (i) Write in terms of the derivatives of the moment generating function (mgf), expressions for the mean and variance of a random variable, X. **[2 marks]**

(ii)$X$is a random variable with moment generating function$m_x(t) = e^{4(t-1)}$. Obtain the mean and the variance of X **[3 marks]**

(iii)Let X be a random variable with probability density function given by$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & otherwise \end{cases}$.Derive the mgf of X **[4 marks]**

(b) A manufacturer supplies DVD players to retailers in batches of 20. 5% of the players are returned because they are faulty

i. Write down a suitable model for the distribution of the number of faulty DVD players in a batch. **[2 marks]**
Find the probability that a batch contains no faulty DVD players **[2 marks]**

ii. Find the mean and variance of faulty DVD players in a batch **[2 marks]**

QUESTION FOUR

(a) Two random variables$x$ and $y$have joint probability distribution as shown:

		x		
y		1	2	3
	1	0.1	0.25	0.35
	2	0.15	0.1	0.05

Find

i. $P(y = 2/x = 3)$ **[1 mark]**

ii. $P(x < 3, y = 2)$ **[2 marks]**

iii. $The\ covariance\ of\ x\ and\ y$ **[2 marks]**

(b) The joint probability density of x and y is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(2x+y) & for \quad 0 < x < 1, \ 0 < y < 2 \\ 0 & elsewhere \end{cases}$$

Find the

i. Marginal density of X **[3 marks]**

ii. Conditional distribution of Y, given that $X = \frac{1}{4}$ **[2 marks]**

(c) A café serves breakfast every morning. Customers arrive for breakfast at random at a rate of 1 every 6 minutes. Find the probability that fewer than 9 customers arrive for breakfast on Monday morning between 10 am and 11 am. **[5 marks]**

QUESTION FIVE

(a) Jean catches a bus to work every morning. According to the time table, the bus is due at 8.00 am but Jean knows that the bus can arrive at a random time between 5 minutes early and 9 minutes late. The random variable X represents the time, in minutes, after 7.55 am when the bus arrives.

i. Suggest a suitable model for the distribution of X and specify it fully. **[2 marks]**

ii. Calculate the mean time of arrival of the bus. **[3 marks]**

iii. Find the cumulative distribution of X. **[4 marks]**

iv. .Jean will be late for work if the bus arrives after 8.05 am. Find the probability that Jean is late for work **[2 marks]**

(b) A discrete random variable Y has the probability distribution as follows:

Y	1	2	3	4	5
$P(Y = y)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

i. Write down the probability generating function G_Y(t) for the random variable Y **[2 marks]**

ii. Find the value of $G_Y(t)$ at $t = 1$ **[2 marks]**

QUESTION SIX

(a) A discrete random variable X has the distribution function, $F(x)$ given by

$$F(x) = \begin{cases} 0 & for\ 0 < -1 \\ \frac{1}{3} & for -1 \le x < 1 \\ \frac{1}{2} & for\ 1 \le x < 3 \\ \frac{5}{6} & for\ 3 \le x < 5 \\ 1 & for\ x \ge 5 \end{cases}$$

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.Find

i. The probability distribution, $f(x)$, of X **[3 marks]**

ii. $P(X \le 3)$ **[2 marks]**

iii. $P(-0.4 < X < 4)$ **[2 marks]**

iv. $Var\ (6X + 11)$ **[4 marks]**

(b) A bag contains 3 red balls and 1 blue ball. A second bag contains 1 red ball and 1 blue ball. A ball is picked from each bag and placed in the other bag. What is the expected number of red balls in the first bag? **[4 marks]**