GARISSA UNIVERSITY COLLEGE

(A Constituent College of Moi University)

UNIVERSITY EXAMINATION 2016/2017 ACADEMIC YEAR ONE SECOND SEMESTER EXAMINATION

SUPPLEMENTARY/SPECIAL EXAMINATION

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: ECO 113

COURSE TITLE: INTRODUCTION TO MATHS II

EXAMINATION DURATION: 3 HOURS

DATE: 28/09/17 **TIME: 2.00-5.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of TWO (2) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Define the following terms:

i. Consumer's surplus **[1 mark]**

ii. **(**Producers' surplus **[1 mark]**

(b) The growing value of the Gross National Product (GNP) is given by

$GNP_t = GNP_0 e^{rt}$ and $r = 1.5\%$

After how many years will the GNP will triple? **[3 marks]**

(c) Find the equilibrium price and quantity for the following single commodity market Model using Cramer's rule **[4 marks]**

$Q_d = 14 - P$

$Q_s = -4 + 0.5P$

$\overline{Q_d} = \overline{Q_s} = \overline{Q}$

(d) The national income determination model for a closed economy is given by

$Y = C + I$

Where $C = C(t)$, $I = I(t)$, $Y = Y(t)$

Find the growth rate of Y if the growth rates of C and I are 3% p.a and 4.5% p.a while the levels of Y, C and I are 200,80 and 120 respectively. **[3 marks]**

(e) Given the total costs (TC) function, $TC = Q^3 - 8Q^2 + 20Q + 15$,compute the level of the output Q at which the total costs are minimized. **[5 marks]**

(f) Find the critical value of the following univariate logarithmic function

$y = In(3x^2 - 12x + 5)$ and confirm that the critical value presents a maximum. **[4 marks]**

(g) Discuss the dynamic stability of the function, $y(t) = 4e^{-2t} + 3$ **[4 marks]**

QUESTION TWO

(a) Given the marginal cost (MC) function, $MC = 15 + 10Q - 6Q^2$ and $TC = 50$ when $Q = 0$.

Find the total cost (TC) function. **[3 marks]**

(b) A firm in a perfectly competitive market has the following demand (P), total variable costs (TVC) and total fixed costs (TFC) functions:

$P = 12.1$

$TVC = \frac{1}{20}Q^3 - 1.5Q^2 + 17.5Q$

$TFC = 50$

Required:

i. Find the total costs (TC), total revenue (TR) and π (profit) function. **[3 marks]**

ii. Find the output level at which profits is maximized **[6 marks]**

iii. Compare the resulting marginal costs (MC) and the marginal revenue (MR) at the profit maximizing point. **[3 marks]**

QUESTION THREE

(a) A firm employing labor as the only factor input has the following production function

$Q = f(L) = Le^{-0.2L}$

Where Q = output and L = labor input

Required:

i. Find the critical value of L. **[2 marks]**

ii. Confirm that the critical value of L maximizes Q. **[5 marks]**

(b) Find the consumers' surplus, given that the demand function, $P = 13 - Q^2$ and equilibrium price, $P_e = 4$. **[4 marks]**

(c) Find the producers' surplus, given that the demand function, $P = 3 + Q^2$ and equilibrium Price, $P_e = 19$ **[4 marks]**

QUESTION FOUR

(a) You are given the following national income model:

$$Y = C + I + G$$

$$C = 120 + 0.8Y$$

$$I = 100 + 0.1Y$$

$$G = 300$$

Required:

i. Present this model in matrix form **[1 mark]**

ii. Using Cramer's rule**,** find the endogenous variables $\overline{Y}, \overline{C}$ and $\overline{I}$. **[6 marks]**

(b) A multinational corporation produces a wide range of electronic products.

P_1 represents the profits from the sales of a new DVD player.

P_2 represents profits from the sales of a new plasma TV set.

P_3 represents profits from the sales of Hifi system.

The economics department believes that the profits in $ are linked as follows:

$$P_1 + 2P_2 + P_3 = 40000$$

$$3P - 4P_2 - 2P_3 = 20000$$

$$5P_1 + 3P_2 + 5P_3 = -10000$$

Required:

Work out the profits of each product using matrix inverse and interpret your answers. **[8 Marks]**

QUESTION FIVE

(a) Consider the following market model:

$$\frac{dP}{dt} = \gamma\left[D(P) - S(P)\right]$$

$$D(P) = 4 - 0.2P$$

$$S(P) = -2 + 0.3P$$

Where $\gamma = \frac{1}{2}$, $D(P) =$ Demand Function, $S(P) =$ Supply Function. Given that $t = 0$

When $P(0) =$ 25,

Required:

i. Form a differential equation from the given equations. **[2 marks]**

ii. Find the general solution to the differential equation. **[6 marks]**

iii. Find the unique solution to the differential equation. **[2 marks]**

(b) The price elasticity of demand is given by

$$\varepsilon_{Q,P} = \frac{-(7P + 4P^2)}{Q}$$

And $Q = 800$ when $P = 5$.

Required:

Construct a differential equation and solve the equation to obtain a demand function. **[5 marks]**

QUESTION SIX

A land speculator has a piece of land whose value grows according to the following function

$$V = 100e^{\sqrt[3]{t}}$$

Where: $V =$ the value of the land at time "t"

$t =$ time

100= the value of the land at time $t = 0$

Required:

Given a discount rate of 8 % under continuous compounding, determine the time over which he must keep the land so as to maximize its present value? Check the second order condition. **[15 marks]**