



GARISSA UNIVERSITY COLLEGE

(A Constituent College of Moi University)

**UNIVERSITY EXAMINATION 2016/2017 ACADEMIC YEAR ONE
SECOND SEMESTER EXAMINATION**

SUPPLEMENTARY/SPECIAL EXAMINATION

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: MAT 104

COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

EXAMINATION DURATION: 3 HOURS

DATE: 28/09/17

TIME: 09.00-12.00 PM

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of FOUR (4) printed pages

please turn over



QUESTION ONE (COMPULSORY)

- (a) State whether the locus traced by the path given by the equation defines a circle in \mathbb{R} : $x^2 + y^2 + 2x + 12y + 112 = 0$ [3 Marks]
- (b) Solve for x between 0° and 180° in the equation $2\tan x = 1 + \cot x$ [3 Marks]
- (c) (i) In an election, a voter may vote for any number of candidates not greater than the number chosen. There are seven candidates and four members to be chosen. In how many ways can a person vote [4 Marks]
- (ii) 4 men and 3 women are to be seated for a dinner such that no 2 women sit together. Find the number of ways in which this can be arranged [3 Marks]
- (d) Using an illustrated diagram, describe briefly what you understand by the following terms as used in conic sections: parabola, focus and directrix [3 Marks]
- (e) Find the equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ joining the points whose eccentric angles are θ, ϕ [5 Marks]
- (f) Obtain the equation of the circle which passes through the points (1,2), (2,3) and (4,0) [4 Marks]

QUESTION TWO

- (a) Given that $\sin(x + \alpha) = \sqrt{2} \cos(x - \alpha)$ show that $\tan x = \frac{\sqrt{2}\tan\alpha}{1 - \sqrt{2}\tan\alpha}$ Hence solve the equation $\sin(x + \pi/6) = \sqrt{2} \cos(x - \pi/6)$ for $0 \leq x \leq 2\pi$ [7 marks]
- (b) Prove the trigonometric identity: $\tan A + \cot A = 2\operatorname{cosec} 2A$ [3 marks]
- (c) Eliminate θ given that $x = \sec\theta + \tan\theta$; $y = \sec\theta - \tan\theta$ [5 marks]



QUESTION THREE

- (a) Show that if x is small enough for its cube and higher powers to be neglected, $\sqrt{\frac{x-1}{x+1}} = 1 - x + \frac{x^2}{2}$. Hence or otherwise, by putting $x = \frac{1}{8}$ show that $\sqrt{7} \cong 2 \frac{83}{128}$ [6 Marks]
- (b) Prove that $2\text{Cosh}^2\theta - 1 = \text{Cosh}2\theta$ [3 Marks]
- (c) Solve the hyperbolic equation $5\text{Cosh}x + \text{Sinhx} = 7$ [3 Marks]
- (d) Express $\text{Sinh}^{-1}x$ in logarithmic form and hence find the value of $\text{Sinh}^{-1}1.534$ [3 Marks]

QUESTION FOUR

- (a) (i) State without proof, the factor theorem [1 Mark]
- (ii) Solve the polynomial equation $2x^3 + 3x^2 - 32x + 15 = 0$ [4 Marks]
- (b) Show that there are 136 ways of selecting 4 letters from the word "EXAMINATION" [3 Marks]
- (c) Describe the motion of the particle whose position $P(x, y)$ at a time t is given by $x = \sec t, y = \tan t, -\frac{\pi}{2} < t < \frac{\pi}{2}$ [3 Marks]
- (d) Show that the distance of a point $P(x_1, y_1)$ to a line $ax + by + c = 0$ in a Cartesian plane is given by:

$$r = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|. \quad [4 \text{ Marks}]$$



QUESTION FIVE (15 MARKS)

- (a) Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ [3 Marks]
- (b) Discuss the parabola $x^2 - x = 6 - 3y$ [5 Marks]
- (c) (i) Find the equation of the tangent to the curve $4x^2 + 9y^2 = 36$ at the point $(1, \frac{4}{3}\sqrt{2})$ [4 Marks]
- (ii) What is the tangent of the acute angle between the pair of lines whose equations are $3y = x - 7$ and $2y = 3 - 4x$ [3 Marks]

QUESTION SIX

- (a) Prove the Binomial Theorem $(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$. [7 Marks]
- (b) Find a complete graph of $r = \frac{6}{4-3\cos\theta}$. Specify a directrix and a range for θ that produces a complete graph. Find the standard form for the equation of the conic. [8 marks]

