GARISSA UNIVERSITY COLLEGE

(A Constituent College of Moi University)

UNIVERSITY EXAMINATION 2016/2017 ACADEMIC YEAR ONE SECOND SEMESTER EXAMINATION

SUPPLEMENTARY/SPECIAL EXAM

SCHOOL OF EDUCATION, BIOLOGICAL AND PHYSICAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: MAT 111

COURSE TITLE: GEOMETRY AND ELEMENTARY APPLIED MATHEMATICS

EXAMINATION DURATION: 3 HOURS

DATE: 26/09/17 **TIME: 09 .00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of FOUR (4) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Define the following terms as used in Geometry:
  i. a circle
  ii. eccentricity, e of an ellipse
  iii. the conjugate axis of a hyperbola
  iv. the dot product of two vectors$\boldsymbol{u}$ and $\boldsymbol{v}$
  v. the vector projection $\boldsymbol{u}$ onto $\boldsymbol{v}$. **[5 marks]**

(b) A car moving with constant acceleration covers the distance between two points 200m in 10seconds.Its speed as it passes the second point is 80km/h. find its speed at the first point and the acceleration of the car. **[3 marks]**

(c) With the help of a sketch diagram, compute the distance from a point $S(1,1,3)$ to the plane given by the equation $x - 2y + 6z = 6$. **[6 marks]**

(d) Find the angle between the planes $6x + 6y - 3z = 5$ and$x - 2y + 2z - 4 = 0$. (4marks)

(e) Describe the motion of a particle whose position $P(x, y)$ at a time t is given by $x = acost, y = bsint$, $0 \leq t \leq 2\pi$ **[4 marks]**

(f) Express in polar co-ordinates the position $(-5,2)$ **[3 marks]**

QUESTION TWO

(a) Prove that the standard form of an equation of an ellipse, with centre (h, k) and major and minor axes of lengths $2a$ and$2b$ respectively, where $a > b$ is given by

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$ **[10 marks]**

(b) Analyze the graph of the equation $4x^2 - 3y^2 + 8x + 16 = 0$. **[5 marks]**

QUESTION THREE

(a) Prove that the angle between two vectors $\boldsymbol{u}$=$\langle u_1, u_2, u_3\rangle$ and $\boldsymbol{v}$=$\langle v_1, v_2, v_3\rangle$ isgiven by

$$\theta = cos^{-1}\frac{(u_1v_1+u_2v_2+u_3v_3)}{|\boldsymbol{u}|\,|v|}$$ **[5 marks]**

(b) Find the area of the triangle PQR with vertices $P(1,2,0), Q(3,0,-3)$ and $R(5,2,6)$ **[5 marks]**

(c) (i) When are three non-zero vectors said to be coplanar? Verify that the vectors $\boldsymbol{a} = (2,3,-1), \boldsymbol{b} = (1,-1,3)$ And $\boldsymbol{c} = (1,9,-11)$ are coplanar. **[3 marks]**

(ii) Find the volume of the parallelepiped determined by $\boldsymbol{u} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{v} = -2\boldsymbol{i} + 3\boldsymbol{k}$ and $\boldsymbol{w} = 7\boldsymbol{j} - 4\boldsymbol{k}$. **[2 marks]**

QUESTION FOUR

(a) A force $\boldsymbol{F} = 2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ is applied to a spacecraft with velocity $\boldsymbol{v} = 3\boldsymbol{i} - \boldsymbol{j}$ .Express **F** as a sum of a vector parallel to $\boldsymbol{v}$ and a vector orthogonal to $\boldsymbol{v}$. **[4 marks]**

(b) Find the symmetric equations for the line in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect **[5 marks]**

(c) i.Given a line L in space and a point P not on L, let $\boldsymbol{m}$ be any parallel vector to L and let Q be any point on L, prove that the shortest distance between P and L is given by

$$d = \frac{|\boldsymbol{m} \times \boldsymbol{QP}|}{|\boldsymbol{m}|}$$ **[2 marks]**

ii. Using results in c (i) above, find the distance between the point $P(4,2,-2)$ and the line L with parametric equations $x = 3 - 2t, y = 6t\ , z = -1 + 9t$. **[4 marks]**

QUESTION FIVE

(a) Show that the area of a plane figure bounded by the polar curve $r = f(\theta)$ and the radius vectors at $\theta = \theta_1$ and $\theta = \theta_2$ is given by $A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$. **[4 marks]**

(b) Find the total area enclosed by the curve $r = 2cos3\theta$. **[4 marks]**

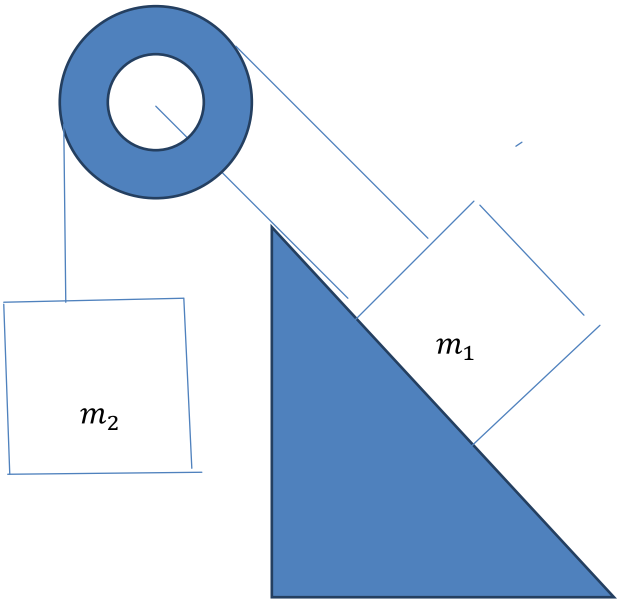
(c) Find the surface area generated when the arc of the curve $r = 5(1 + cos\theta)$ between $\theta = 0$ and $\theta = 2\pi$, rotates completely about the initial line. **[7 marks]**

QUESTION SIX

(a) Find a complete graph of $r = \frac{6}{4-3cos\theta}$ .Specify a directrix and a range for θ that produces a complete graph. Find the standard form for the equation of the conic.

[7 marks]

(b) A block of mass m_1 lying on an inclined plane is pulled up by a mass m_2, the two masses being connected by a light inextensible cord passing over a smooth pulley. Given that the coefficient of static friction between m_1 and the plane is 0.15, and that $m_1 = m_2 = 2.0kg$, determine the acceleration of the masses for a plane inclined at $30°$ to the horinzotal. **[8 marks]**


m_1
m_2