GARISSA UNIVERSITY COLLEGE

(A Constituent College of Moi University)

UNIVERSITY EXAMINATION 2016/2017 ACADEMIC YEAR <u>ONE</u> <u>SECOND</u> SEMESTER EXAMINATION

<u>SUPPLEMENTARY/SPECIAL EXAM</u>

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION, Bsc AND Bsc (COMPUTER SCIENCE)

COURSE CODE: MAT 113/ MAT110

COURSE TITLE: DIFFERENTIAL CALCULUS/ BASIC CALCULUS

EXAMINATION DURATION: 3 HOURS

DATE: 25/09/17 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of THREE (3) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Evaluate the following limits:

i. $\lim_{x\to 2}\left\{\frac{x^2+x-6}{2x^2-8}\right\}$ **[3 Marks]**

ii. $\lim_{n\to\infty}\left\{\frac{(8x-1)(2x+1)}{(2x-1)(x+1)}\right\}$ **[4 Marks]**

(b) Find the derivatives of the following:

i. $y = (2x^4 - 3x)^5$ **[3 Marks]**

ii. $y = e^{3x}\ln 2x$ **[3 Marks]**

iii. $y = tan^{-1}x$ **[3 Marks]**

(c) Find the equation of the normal to the curve at point$y = x^3 + 3x^2 - 2x - 3$at (1,-1) **[4 Marks]**

(d) A ladder 10 meters long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at a rate of 3 meters per second. How fast is its height on the wall decreasing when the foot of the ladder is 6 meters away from the wall? **[5 marks]**

QUESTION TWO

(a) Find the interval in which the function $f(x) = x^3 - 6x^2 + 3x + 1$ is concave up and concave down. **[4 Marks]**

(b) Find a point on the graph $y = x^3$ where the tangent to the chord joining $(1,1)$ and (3,27). **[5 Marks]**

(c) Given that $f(x) = \frac{ax+b}{x+1}$,$\lim_{x\to 0}\{f(x)\} = 2$ and $\lim_{x\to\infty}\{f(x)\} = 1$, find the value of $f(-3)$ **[6 Marks]**

QUESTION THREE

(a) Differentiate $y = x^2$ from the definition of a derivative or using the first principles. **[3 Marks]**

(b) Find the gradient of the curve $x = \frac{t}{1+t}$, $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2},\frac{1}{2}\right)$ **[6 Marks]**

(c) Find the equation of the tangent and normal to the curve $x^3 + x^2y + y^3 - 7 = 0$ at the point $x = 2, y = 1$ **[6 Marks]**

QUESTION FOUR

(a) The perimeter of a triangle is 8cm. If one of the sides is 3cm, what are other the other two sides for maximum area of the triangle **[4 Marks]**

(b) Find the stationary points of the function $y = x^3 - 5x^2 + 3x + 2$ and distinguish them. **[7 Marks]**

(c) Find the point of inflexion on the graph of the function $y = x^4 - 54x^2 - 2x$. **[4 Marks]**

QUESTION FIVE

(a) Differentiate

(i) $y = x^x$ **[3 Marks]**

(ii) $y = \dfrac{x^2 \sin x}{\cos 2x}$ **[4 Marks]**

(b) A window is in the form of a rectangle, surmounted by a semi-circle. If the perimeter of the window is to be 20 meters, find the dimensions so that the greatest possible amount of light may be admitted **[8 Marks]**

QUESTION SIX

(a) Differentiate $y = a^x$ **[3 Marks]**

(b) Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in the interval $[0,4]$ and find c . **[5 Marks]**

(c) Verify Rolle's Theorem for the function $f(x) = e^{2x}(x^2 - 4x + 3)$ on $[1,3]$ **[7 Marks]**